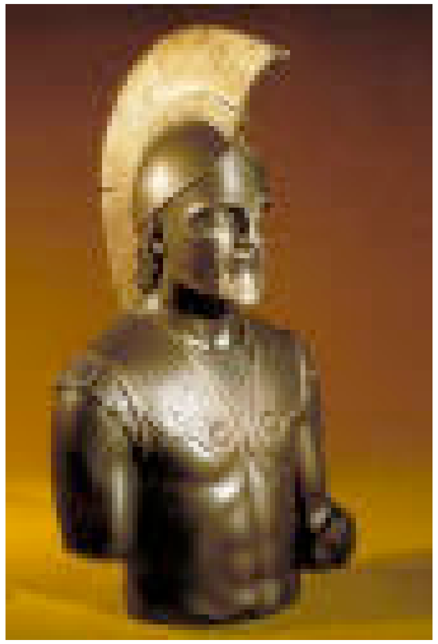
Helden

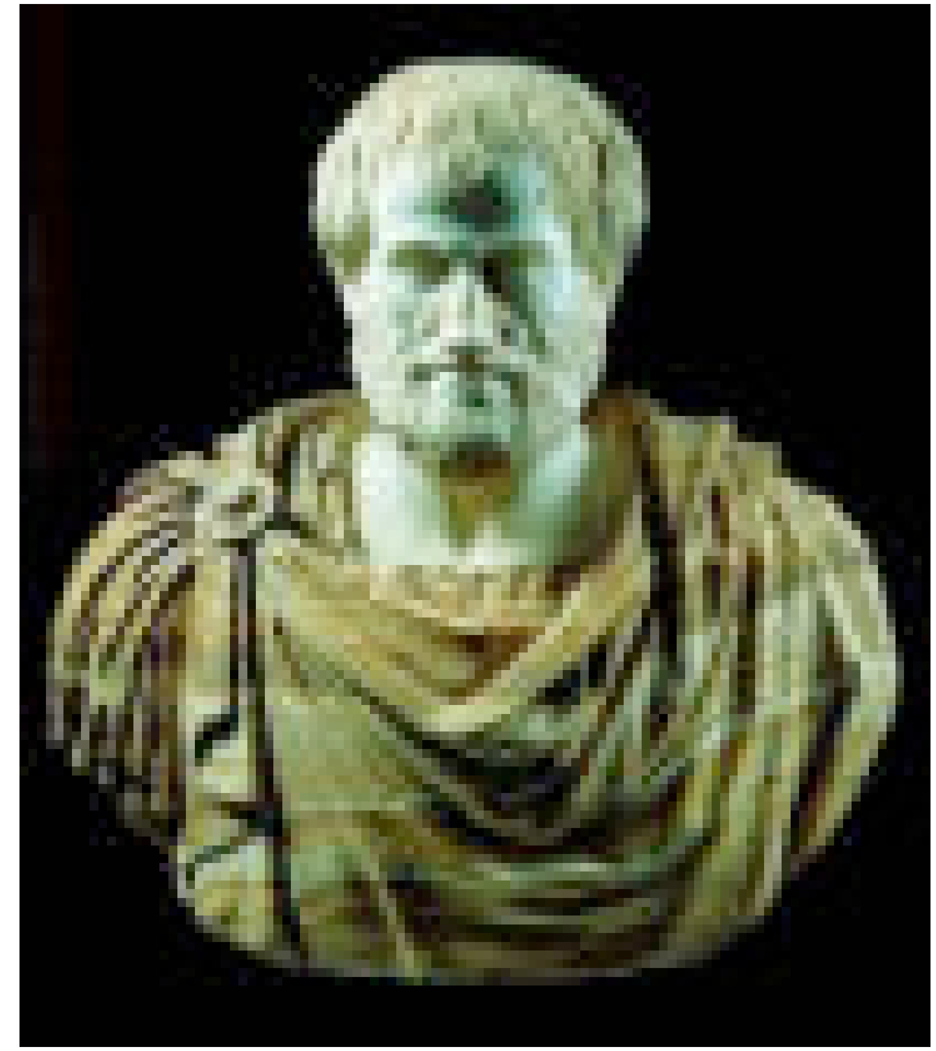


in de

Toegepaste Logica

’Αριστοτελησ

384 vC- 322 vC

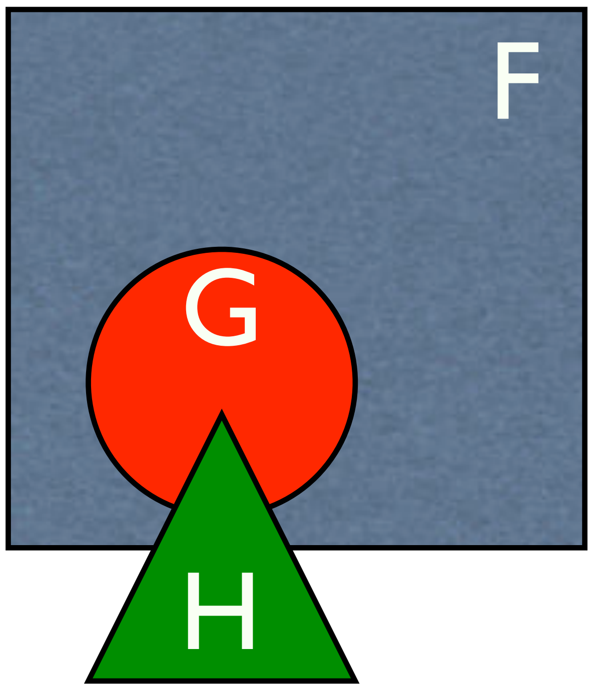


24 logische syllogismen, fragment van predikatenlogica

I. Barbara, Celarent, Darii, Ferio, Barbari, Celaront
II. Cesare, Camestres, Festino, Baroco, Cesaro, Camestros
III. Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison
IV. Bamalip, Calemes, Dimatis, Fesapo, Fresison, Calemos

DIMARIS

sommige H zijn G
elke G is F
sommige F zijn H



BA

Trivium:
grammatica, rhetorica, logica

MA

Quadrivium:
rekenkunde, meetkunde, astronomie, harmonieleer

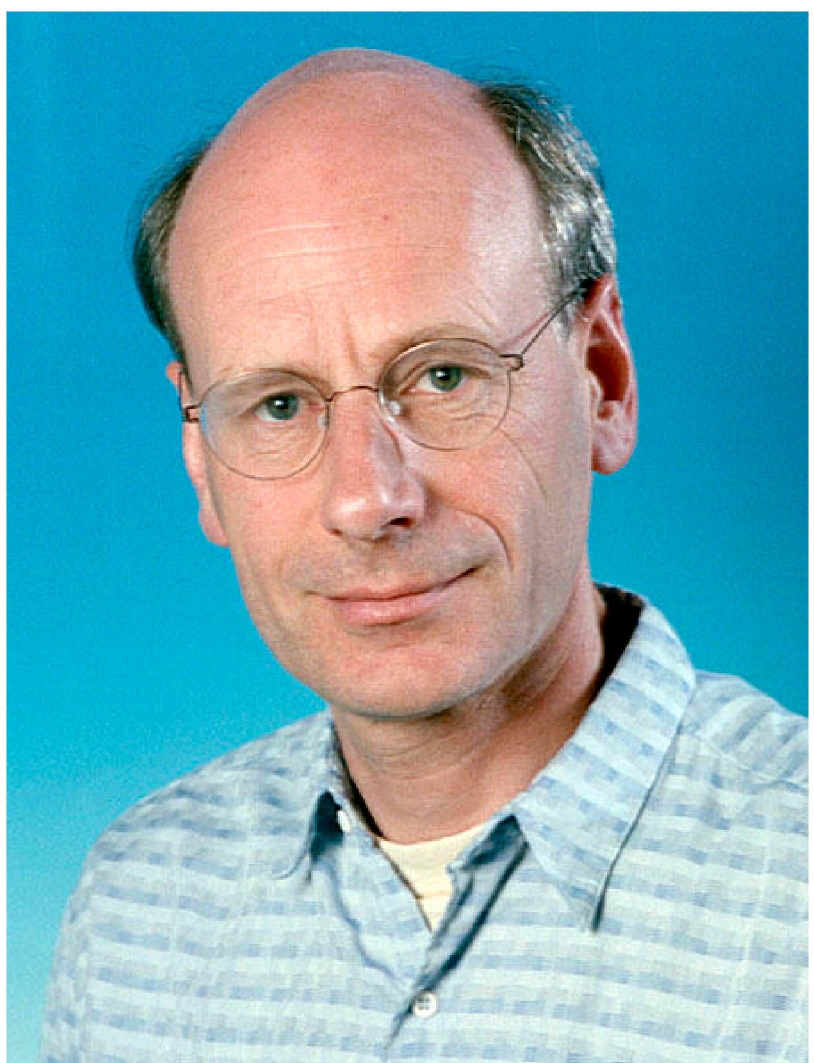
Gottfried Leibniz

1646 - 1716

calculus ratiocinator

Leibniz equality:

in proof checker Coq



- binaire getallen, ...
- calculus ratiocinator: general system of a notation in which all the truths of reason should be reduced to a calculus. Een ‘algebra van gedachten’.
- projectvoorstel “I think that some chosen men could finish the matter within five years”
- Geen aio’s - I had been less busy, or if I were younger or helped by well-intentioned young people, I would have hoped to have evolved a characteristic of this kind

Bernhard Bolzano

1781 - 1848

Paradoxien des Unendlichen

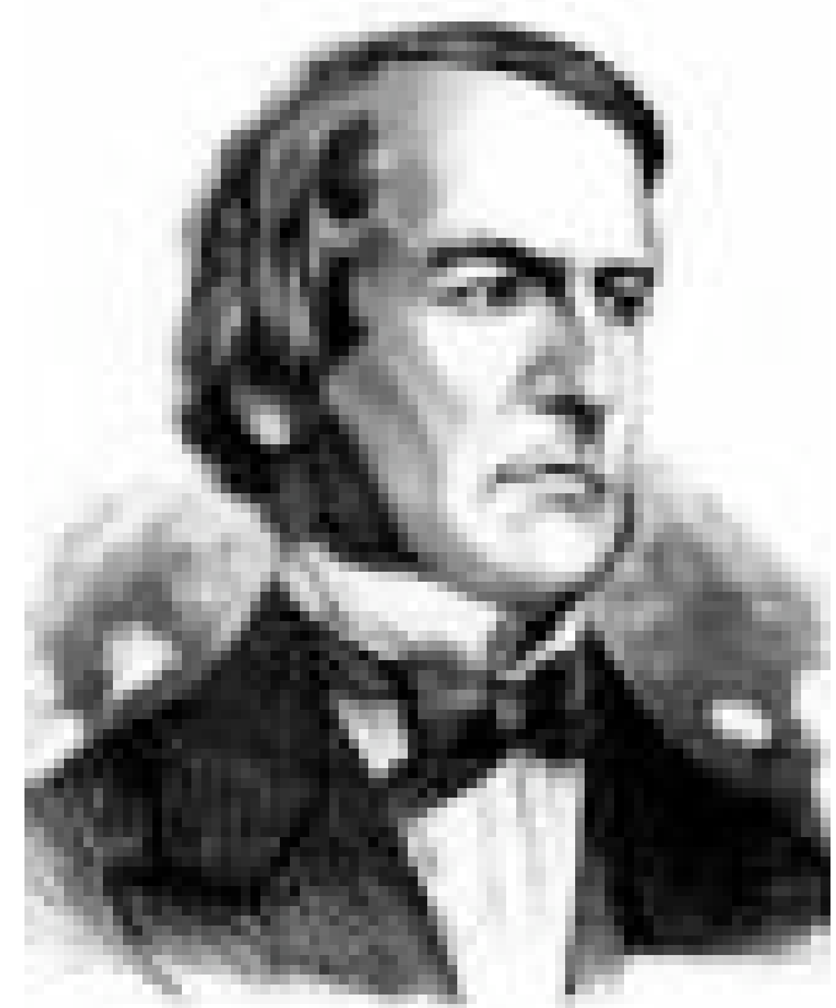
disputed, to be sure, that addends determine their sum, and that equal addends yield equal sums. This holds not only for finite but also for infinite sets of summands. In the case of the latter, however, it is necessary to make sure that the infinite set of summands in the one sum really is identical with the infinite set of summands in the other sum; seeing, namely, that there are different kinds of infinite set. And to make sure of that point, we see from our theorem how altogether insufficient it is to be able 36
to pair off the terms in the one sum with those in the other. The conclusion will be unsafe unless *the two sets have identical terms of specification* (gleiche Bestimmungsgründe). The sequel will bring many examples of the absurdities in which a calculation with the infinite involves us if we fail to pay attention to this point.

§25

I now proceed to the assertion that there exists an infinite *even in the realm of the actual*, and not merely among the things which make no claim to actuality. Anyone who had arrived at the momentous conviction (whether by a chain of reasoning from purely conceptual truths or otherwise) *that there exists a God*, a Being whose existence is grounded in that of no other being, and precisely for this reason is a *universally perfect Being*, uniting in himself all powers and perfections which are compatible with one another at all, and each of them in the highest degree of which it is capable—such a person, I say, agrees by this very fact upon the existence of a Being possessed of infinitude in more than one respect; with respect to his *knowledge*, in that he *knows infinitely much*, to wit, the sum of all truths; to his *volition*, in that he *wills infinitely much*, to wit, the sum of every single possible good; and to his might, or *action ad extra*, in that he *confers actuality*, in virtue of his power of action ad extra, to *everything that he wills*. From this last attribute of God follows the existence of beings other than God, *creatures*, which we contrast with him and call merely *finite beings*, but in which for all that many a trace of infinitude can be found. For the *set* of such beings must 37
already be an infinite one, as also the set of all the *conditions* experienced by any single one of them during no matter how short an interval of time—because every such interval contains infinitely many instants. We therefore encounter infinites even in the realm of the actual.

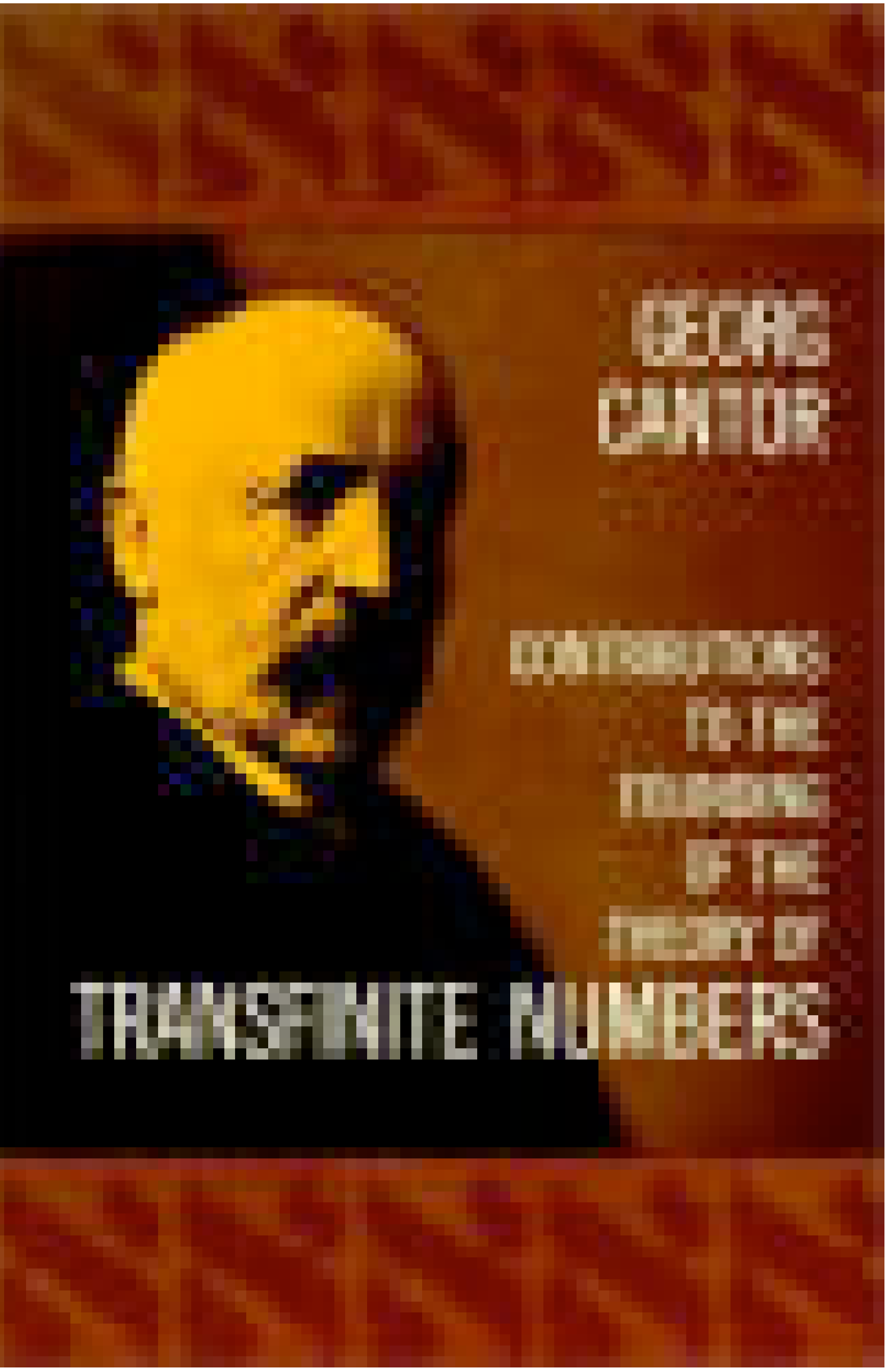
George Boole

1815 - 1864



Georg Cantor

1845 - 1918



I see it, but I don't believe it!

$$\aleph_0, \aleph_1, \aleph_2, \aleph_3, \ldots, \aleph_{1000}, \ldots$$

$$\aleph_{\aleph_0}$$

$$\aleph_{\aleph_{\aleph_0}}$$

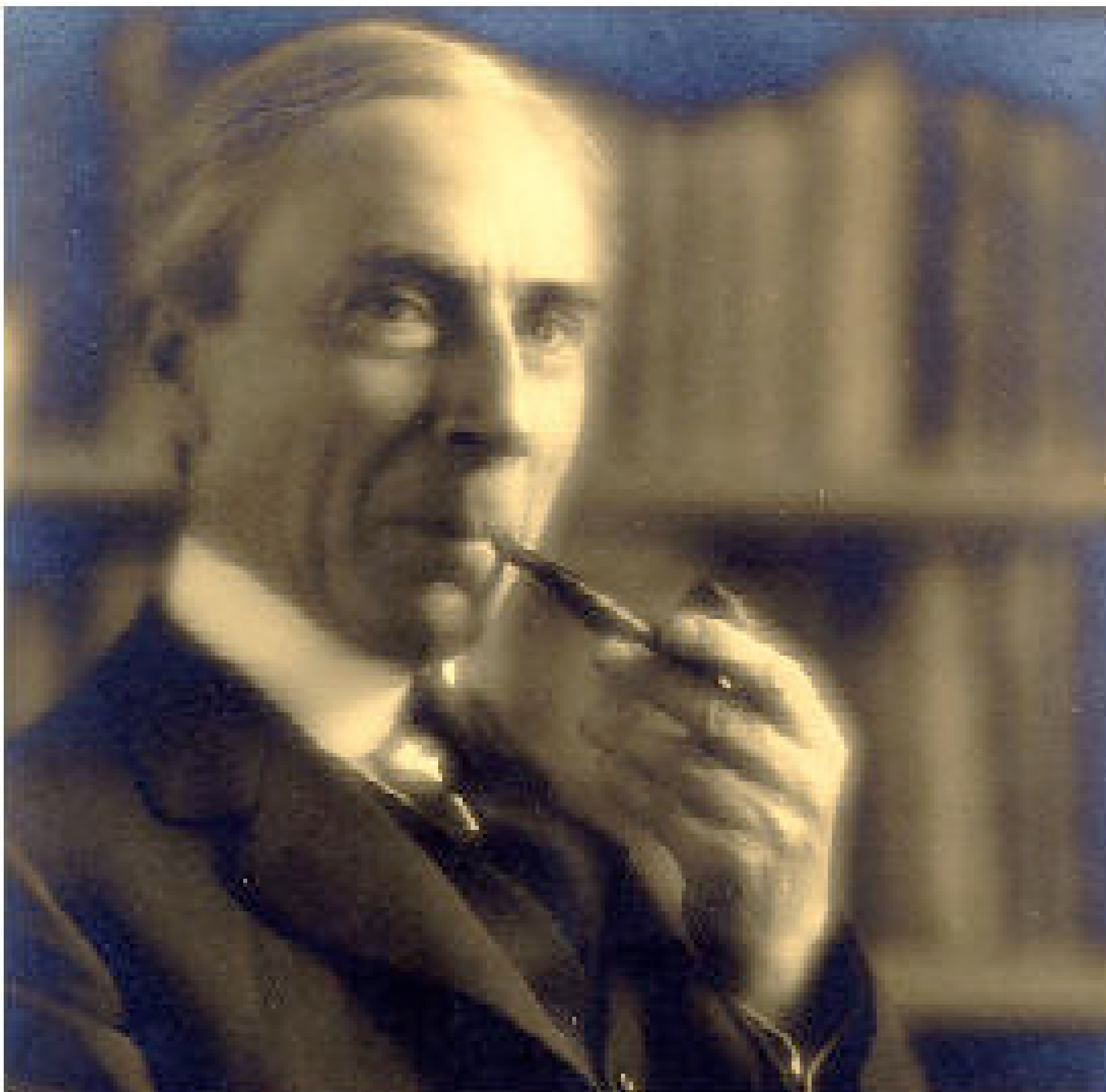
$$\aleph_{\aleph_{\aleph_{\aleph_0}}}$$

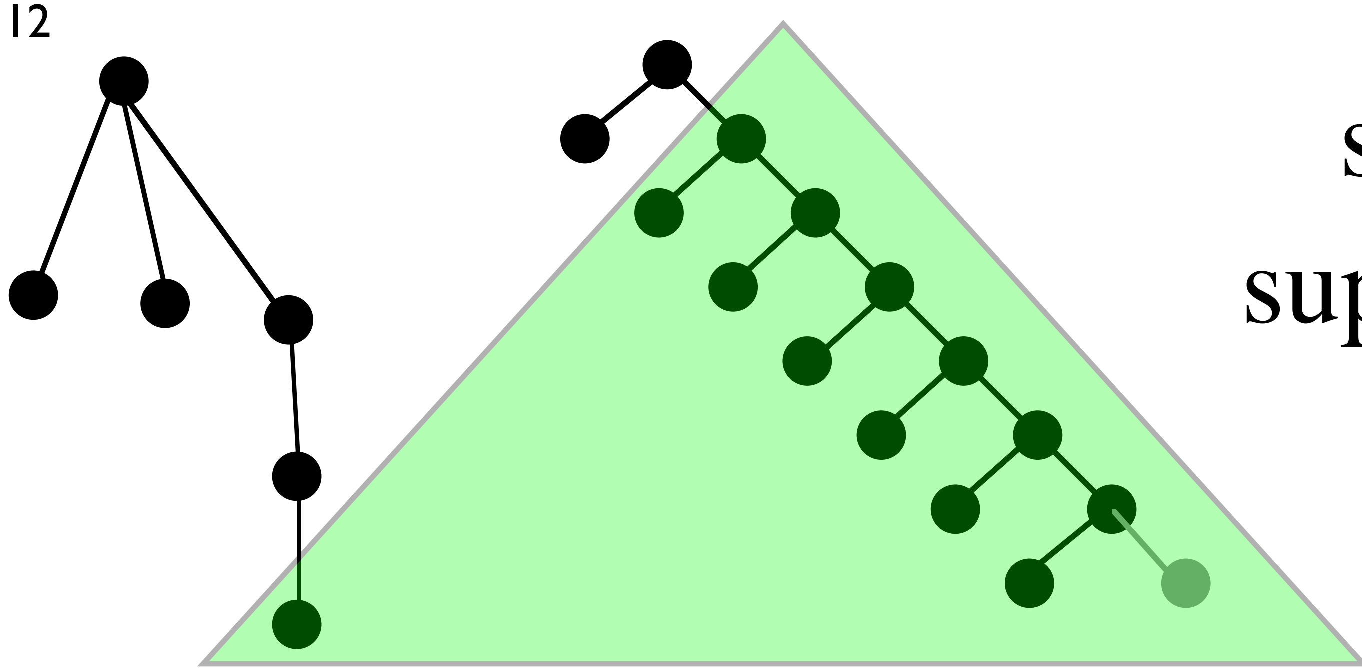
$$\alpha = \aleph_\alpha$$

Ik zag de Aleph, vanuit alle gezichtspunten, ik zag in de Aleph de aarde, en in de aarde weer de Aleph en in de Aleph de aarde, ik zag mijn gezicht en mijn ingewanden, ik zag jouw gezicht, ik voelde een duizeling en huilde, omdat mijn ogen dat geheime, slechts bij gissing bestaande voorwerp hadden gezien, waarvan de naam wederrechtelijk door de mensen gebruikt wordt, maar dat geen mens heeft aanschouwd: het onvoorstelbare heelal.
J.L. Borges, De Aleph.

Gottlob Frege: Begriffschrift

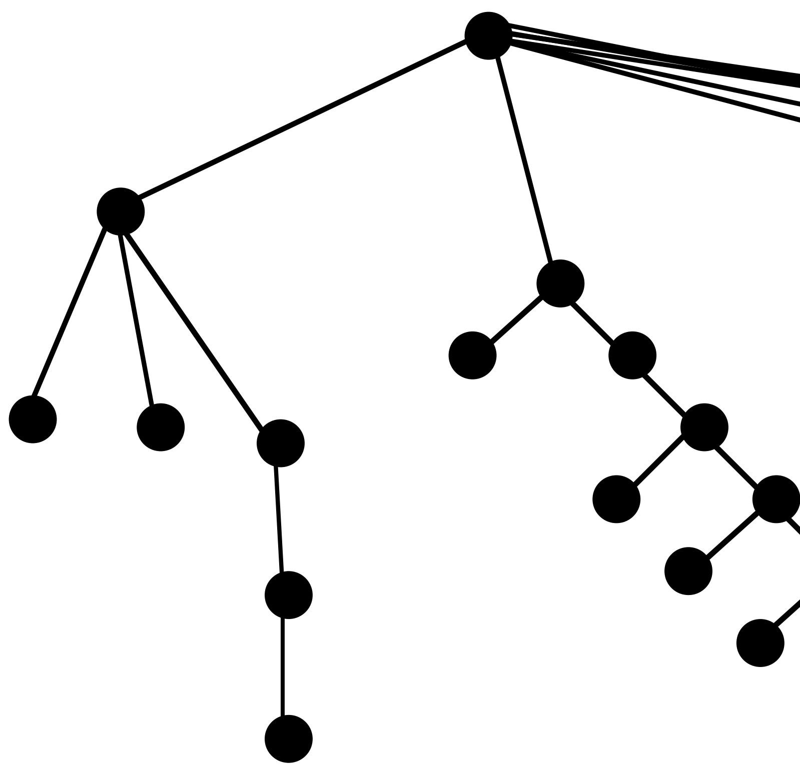
1848 - 1925





superboom is bijzonder ⇒
superboom is niet bijzonder ⇒
superboom is bijzonder

$A \Leftrightarrow \neg A$

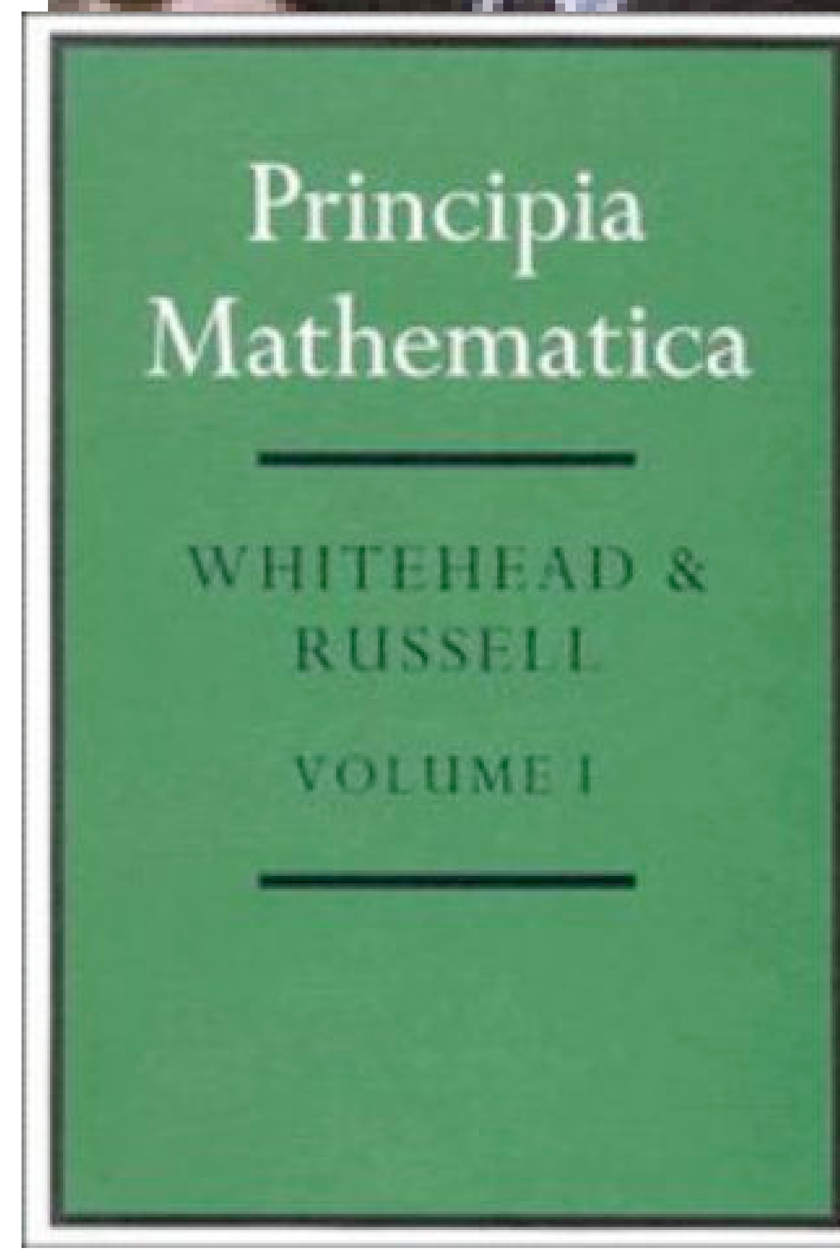


superboom met als di

Bertrand Russell

1872- 1970

Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.



Kurt Gödel

1906- 1978

Either mathematics is too big for the human mind or the human mind is more than a machine.

Jacques Herbrand

1908-1931

logisch programmeren

PROLOG

In 1931, he was awarded a Rockefeller fellowship that enabled him to study in Germany, first with John von Neumann in Berlin, then during June with Emil Artin in Hamburg, and finally, in July, with Emmy Noether.

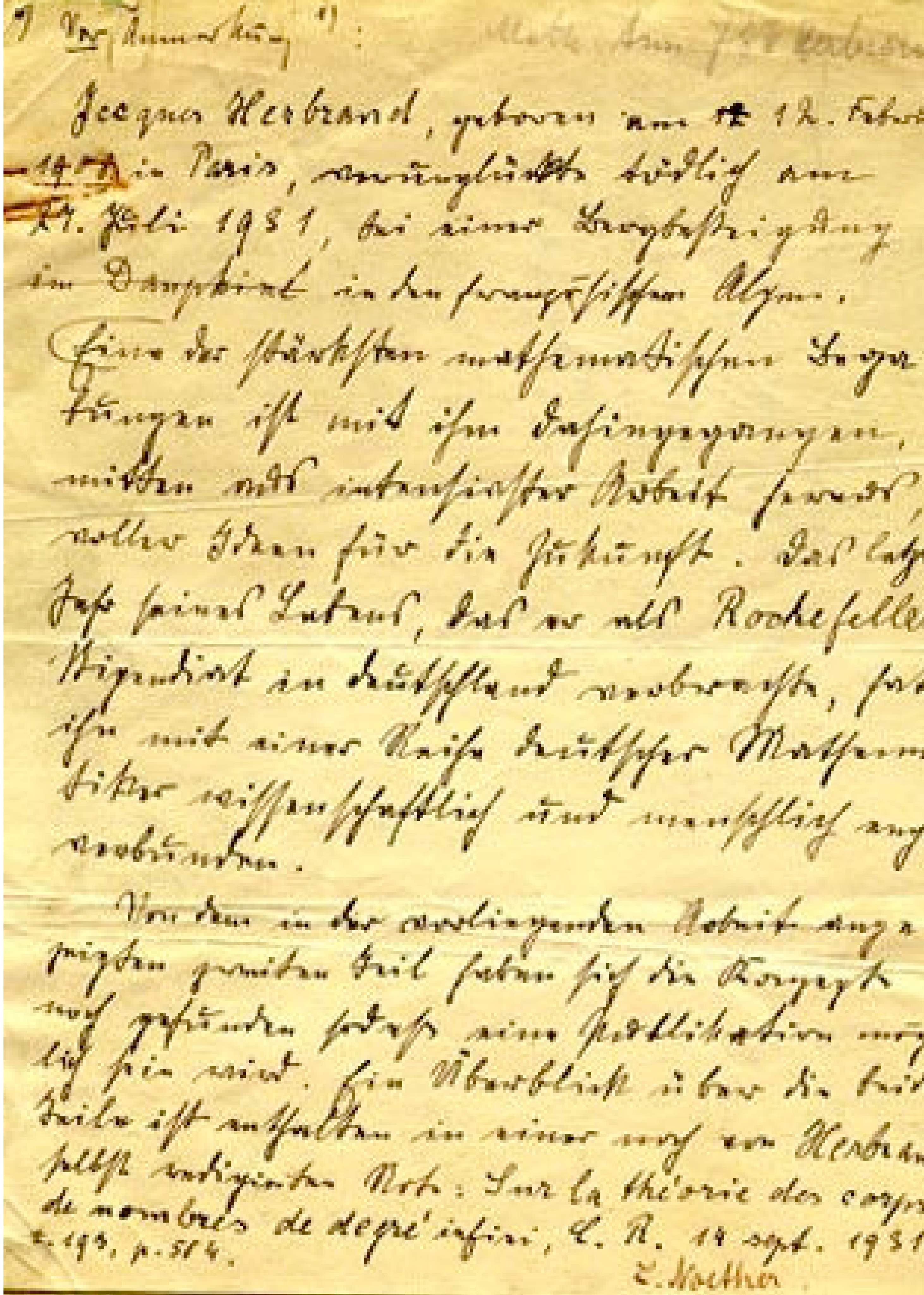
Shown here is a note written in Emmy Noether's hand, sent to an editor of *Mathematische Annalen,* and published as a preface to the article,

Jacques Herbrand, "Théorie arithmétique des corps de nombres de degré infini," *Mathematische Annalen* 106 (1932) 473-501.

Following is a partial translation, based on that given on page 43 of

James W. Brewer and Martha K. Smith, editors, *Emmy Noether: A Tribute to Her Life and Work,* Marcel Dekker, New York, 1981:

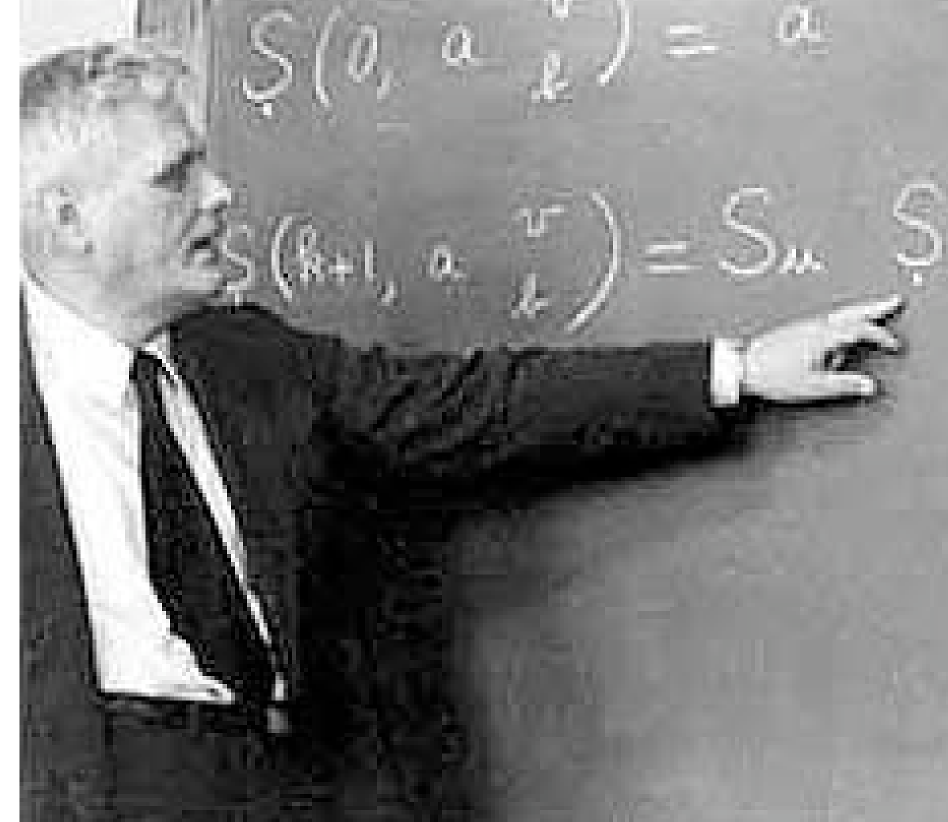
Jacques Herbrand, born on the 12th of February, 1908, in Paris, was killed on the 27th of July, 1931, while mountain-climbing in the French Alps. One of the strongest mathematical talents has passed on with him, just during a time of the most intensive work, while he was full of ideas for the future. The last year of his life, which he spent with a Rockefeller scholarship in Germany, brought him into close contact, both scienfically and personally, with a number of German mathematicians . . .

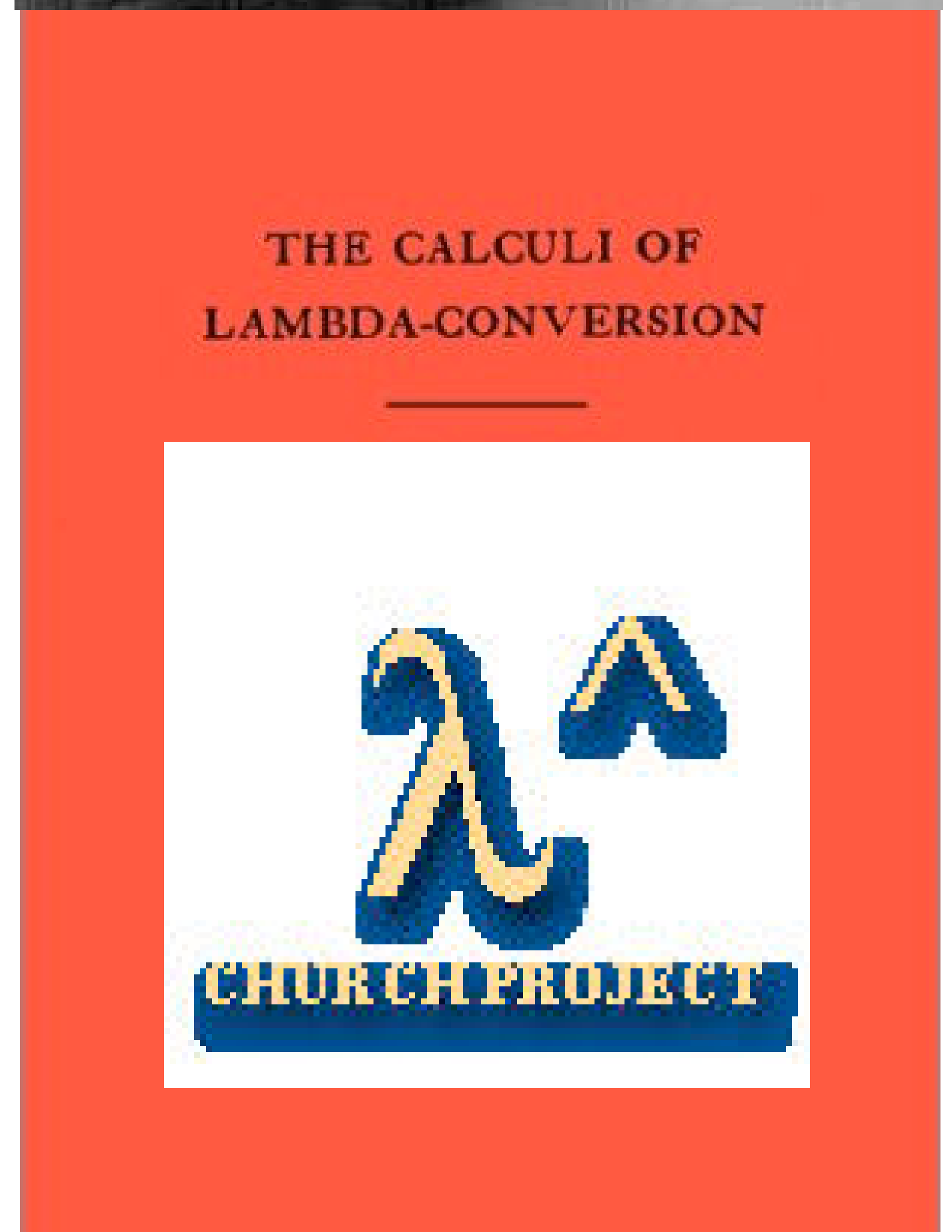

Jacques Herbrand, geboren am 12. Febr
1908 in Paris, verunglückte tödlich am
27. Juli 1931, bei einer Bergbesteigung
im Dauphiné in den französischen Alpen.
Eine der stärksten mathematischen Bega
bungen ist mit ihm dahingegangen,
mitten aus intensivster Arbeit heraus,
voller Ideen für die Zukunft. Das letz
Jahr seines Lebens, das er als Rockefelle
Stipendiat in Deutschland verbrachte, ha
ihn mit einer Reihe deutscher Mathem
tiker wissenschaftlich und menschlich en
verbunden.
Von dem in der vorliegenden Arbeit ange
zeigten zweiten Teil fanden sich die Konzepte
noch gefunden, sodass eine Publikation mög
lich sein wird. Ein Überblick über die bei
Teile ist enthalten in einer noch von Herbra
selbst redigierten Note: Sur la théorie des corp
de nombres de degré infini, C. R. 14 sept. 193
t. 193, p. 504.
E. Noether

Alonzo Church

1903- 1995

At the time of his death, Church was widely regarded as the greatest living logician in the world

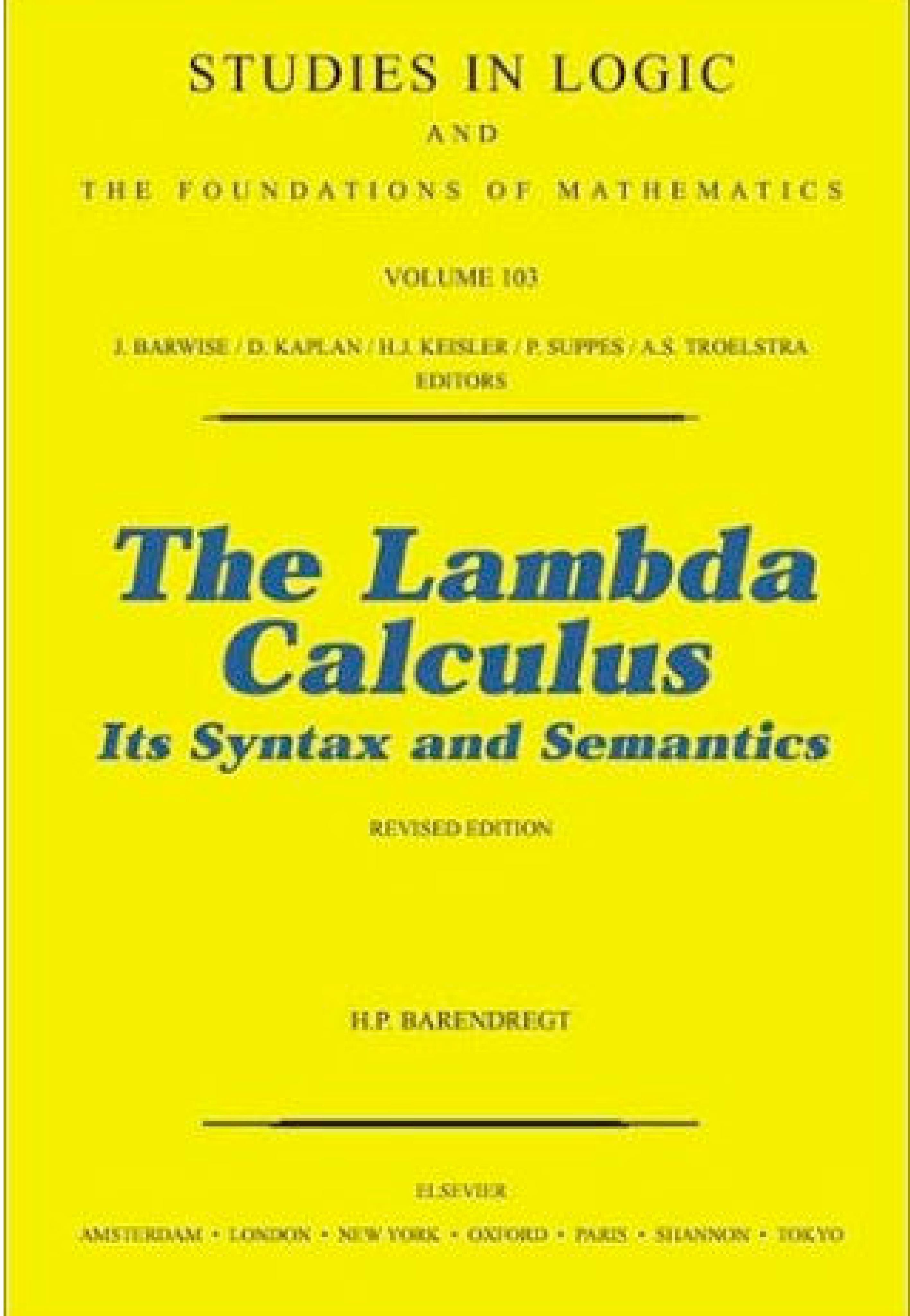




Lambda Calculus

$$(\lambda x.Z(x))Y \longrightarrow Z(Y)$$

Turing compleet



$(\lambda ab.b(aab))(\lambda ab.b(aab))\ M \rightarrow$

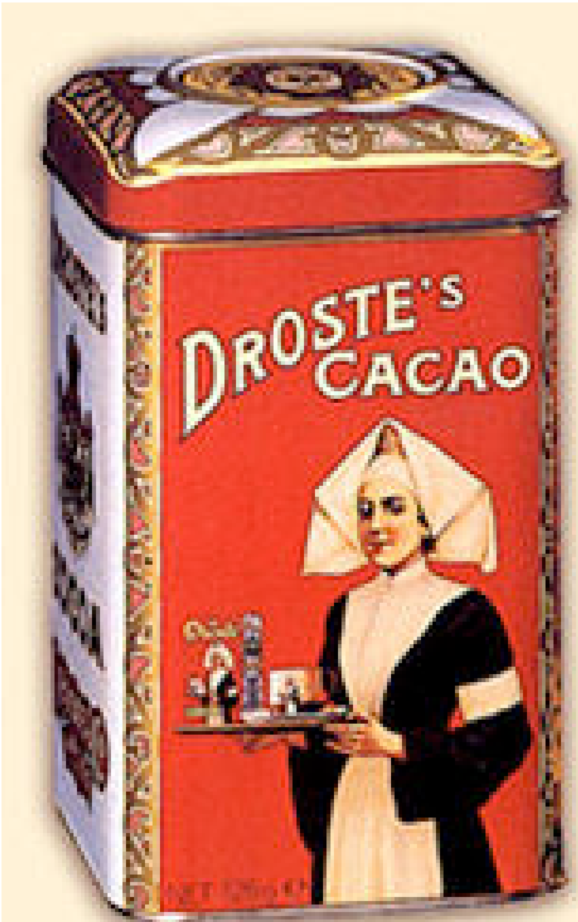
$(\lambda b.b((\lambda ab.b(aab))(\lambda ab.b(aab))b))M \rightarrow$

$M((\lambda ab.b(aab))(\lambda ab.b(aab))M)$

■ $\rightarrow\ \rightarrow M($ ■ $)$

■ $= M($ ■ $)$

■ *is fixed point van M*



Haskell Curry

1900- 1982

A General Substitution Principle in Logical Alg...

A logical operation ϕ shall be called direct ...

$$(a < b) \to (\phi a < \phi b)$$

inverse if $(a < b) \to (\phi b < \phi a)$

Examples of direct operations ϕx are

xc

$c < x$

where c is a particular element of the algebra. (The directness of the first of these follows from AB, that of the second from $(PB)_1$.) Examples of inverse operations, with the postulates from which their inverseness follows are

$x \to c$ (PB_2)

$-x$ $(a < b) \to (-b < -a)$

We consider now operations as composed of a succession of operations ϕ. e.g.

$(a_1 < x) \cdot (a_2 \to (a_3 \to a_4))$ = $\phi_4(\phi_3(x))$

$(a_1 < a_2) \cdot (a_3 \to (x \to a_4))$ = $\phi_4(\phi_3(\phi_2(x)))$

where ϕ_2: $a_3 \to x$

Combinatory Logic

SII(SII)

$$Ix \longrightarrow x$$

$$Kxy \longrightarrow x$$

$$Sxyz \longrightarrow xz(yz)$$

Turing compleet

David Hilbert

1862- 1943

Mathematical Problems

Lecture delivered before the International Congress of Mathematicians at Paris in 1900

By Professor David Hilbert

Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought will the new centuries disclose?

1. Cantor's problem of the cardinal number of the continuum

2. The compatibility of the arithmetical axioms

10. Determination of the solvability of a diophantine equation

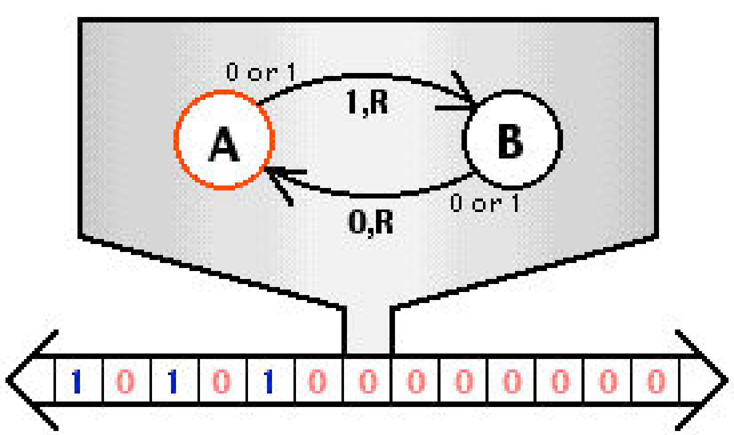
16. Problem of the topology of algebraic curves and surfaces

Alan Turing

1912 - 1945

promoveerde bij von Neumann in 1938

The enigma of intelligence



Alfred Tarski

1901 - 1983

waarheid is ondefinieerbaar

The essence of mathematics lies in its freedom.

Ludwig Wittgenstein

1889 - 1951

Wittgenstein snijdt door alle klassen heen...

Tractatus logico-philosophicus

1
The world is everything that is the case.

7
Whereof one cannot speak, thereof one must be silent.

1

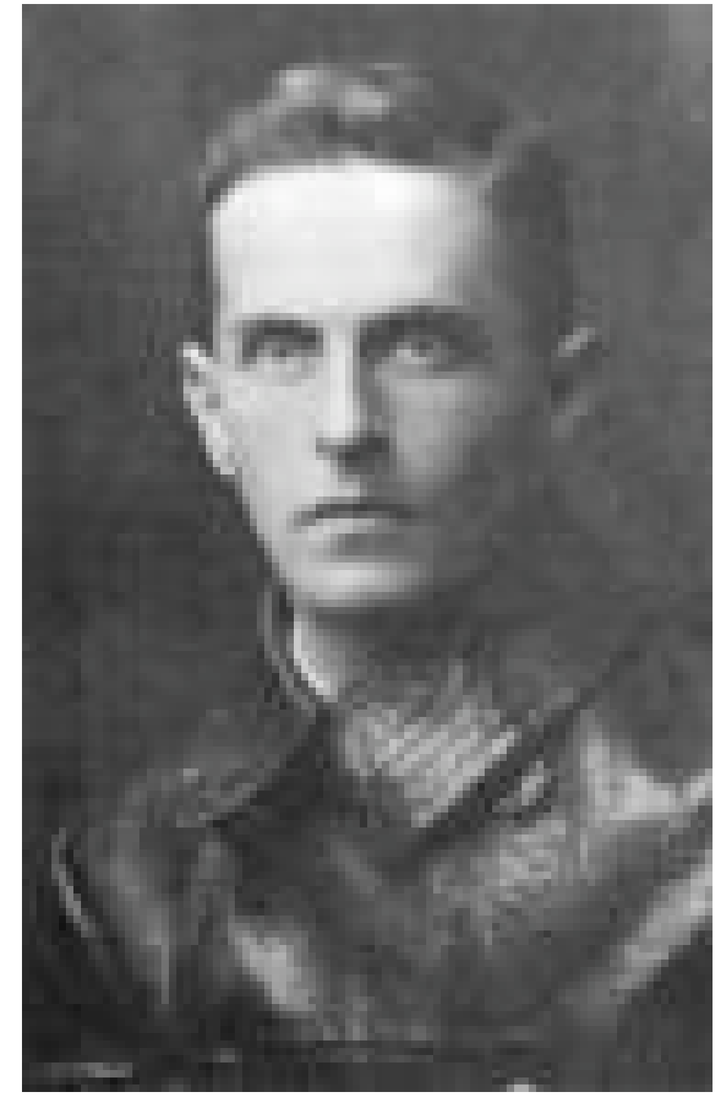
The world is everything that is the case. *

1.1

The world is the totality of facts, not of things.

1.2

The world divides into facts.



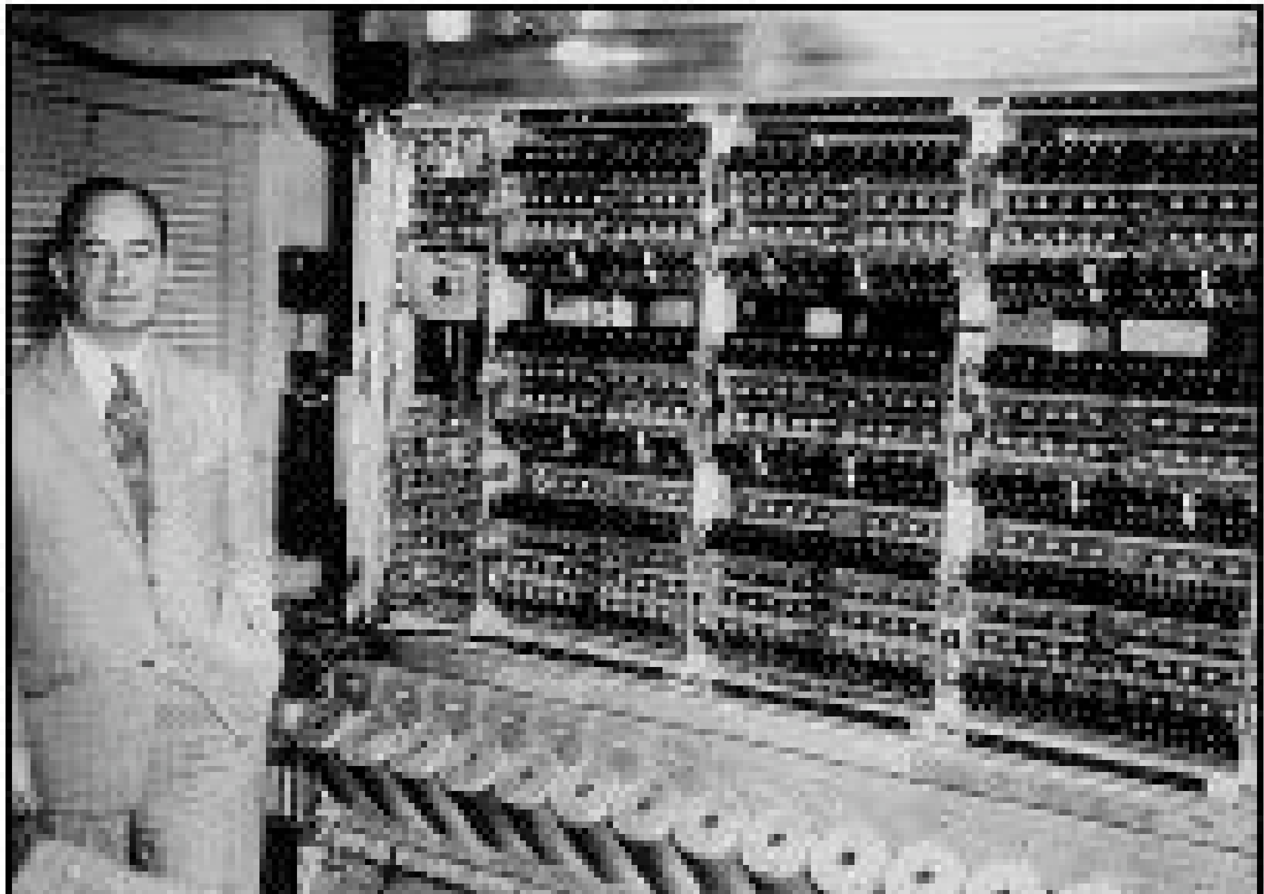
7

Whereof one cannot speak, thereof one must be silent.

John von Neumann

1903 - 1957

game theory, computer architecture, cellular automata, set theory, ordinaalgetallen



Dick de Bruijn

1918

AUTOMATH
ARCHIVE

Institute in Nijmegen and the Formal Methods section of Eindhoven University of Technology. Started by prof. H. Barendregt, in cooperation with Rob Nederpelt, this archive project was launched to digitize valuable historical articles and other documentation concerning the Automath project.

Initiated by prof. N.G. de Bruijn, the project Automath (1967 until the early 80's) aimed at designing a language for expressing complete mathematical theories in such a way that a computer can verify the correctness. This project can be seen as the predecessor of type theoretical proof assistants such as the well known Nuprl and Coq.

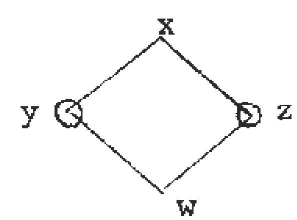
A note on weak diamond properties.

1. Introduction. Let S be a set with a binary relation $>$. We assume it to satisfy $x > x$ for all $x \in S$. We are interested in establishing a property CR (named after its relevance for the Church-Rosser theorem of lambda calculus, cf. [1]). We say that $x \sim y$ if $x > y$ or $y > x$. We say that $x >^{*} y$ if there is a finite sequence $x_1,\ldots,x_n$ with $x=x_1 > x_2 > \ldots > x_n=y$, and also if $x=y$. We say that $(S,>)$ satisfies CR if for any sequence $x_1,\ldots,x_n$ with

$$x_1 \sim x_2 \sim \ldots \sim x_n$$

there exist an element $x \in S$ with both $x_1 >^{*} z$ and $x_n >^{*} z$.

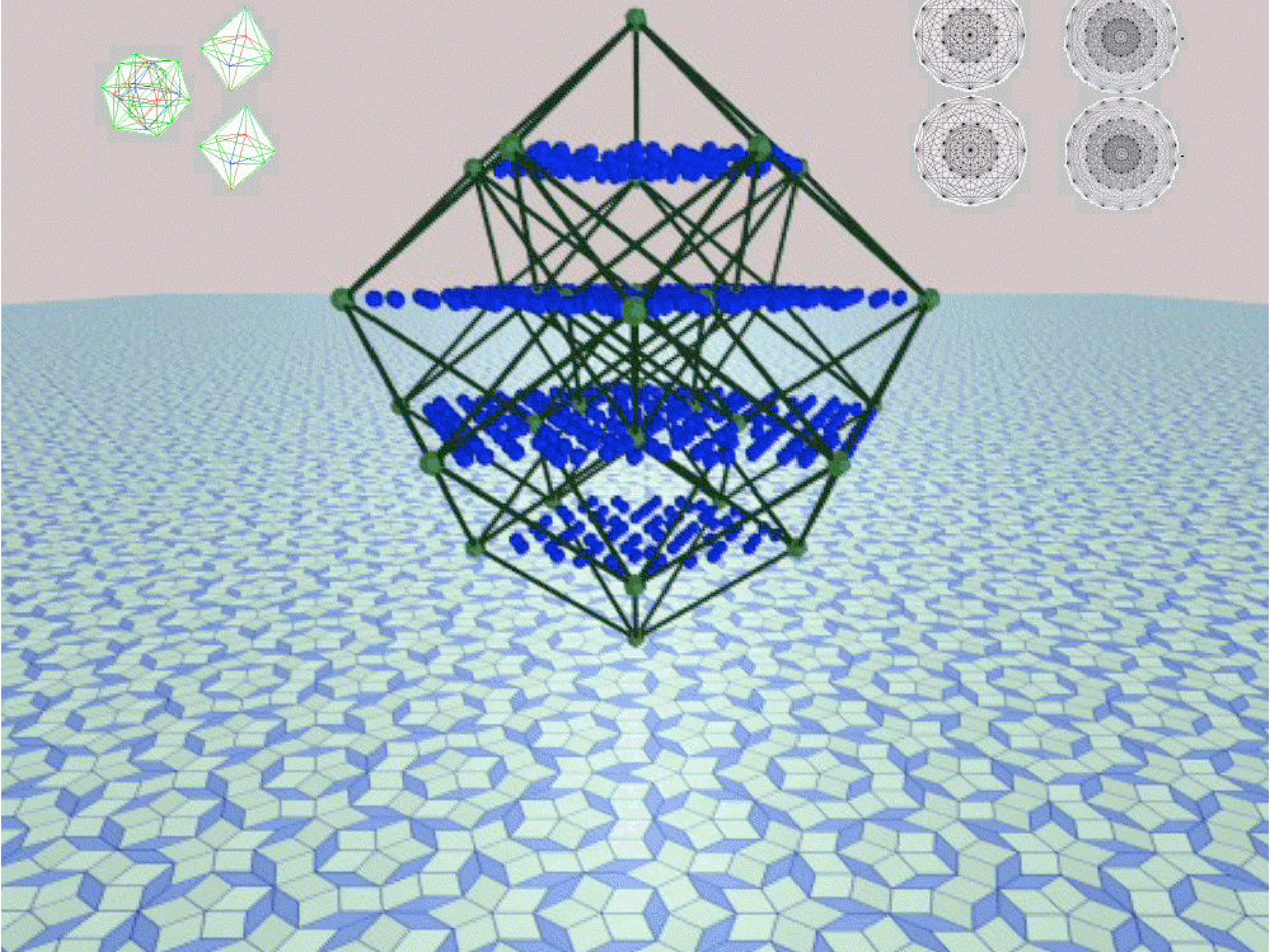
It is usual to say that $(S,>)$ has the diamond property (DP) if for all x,y,z with $x > y$, $x > z$ there exists a w with $y > w$, $z > w$. This is depicted in the following diagram:



where $x > y$ is indicated by a line from x downwards to y, etc. The little circles around y and z illustrate the logical situation: the diagram  can be closed by  .

It is not hard to show that DP implies CR. A simple way to present a proof is by counting "inversions" in sequences like $x_1 > x_2 < x_3 < x_4 > x_5 < x_6 > x_7$: if $i < j$ and $x_i < x_{i+1}$, $x_j > x_{j+1}$, then we say that the pair (i,j) forms an inversion. Applications of DP, like replacing $x_3 < x_4 > x_5$ by $x_3 > x_4^{*} < x_5$, decrease the number of inversions. Once all inversions are gone, we have established CR.

The following property WDP_1 is weaker than DP. It says: "if $x > y$ and $x > z$ then w exists such that $y >^{*} w$ and $z >^{*} w$". It is very frustrating in attemps to prove the Church-Rosser theorem for various systems, that WDP_1 does not imply CR. A counterexample can be obtained by means of the following picture (cf. [2] p. 49):



L.E.J. (Bertus) Brouwer

1881-1966

intuitionisme

~~$A \vee \neg A$~~ *tertium non datur*

~~$\neg\neg A \leftrightarrow A$~~

3, 5, 7/8, ... *rationaal*

e, π, $\sqrt{2}$ *irrationaal*

zijn er α en β, irrationaal,
zodat α^{β} rationaal is?

~~$(\sqrt{2})^{\sqrt{2}}$ is rationaal of irrationaal~~

JA

JA

$\alpha = \beta = \sqrt{2}$:
α^{β} rationaal

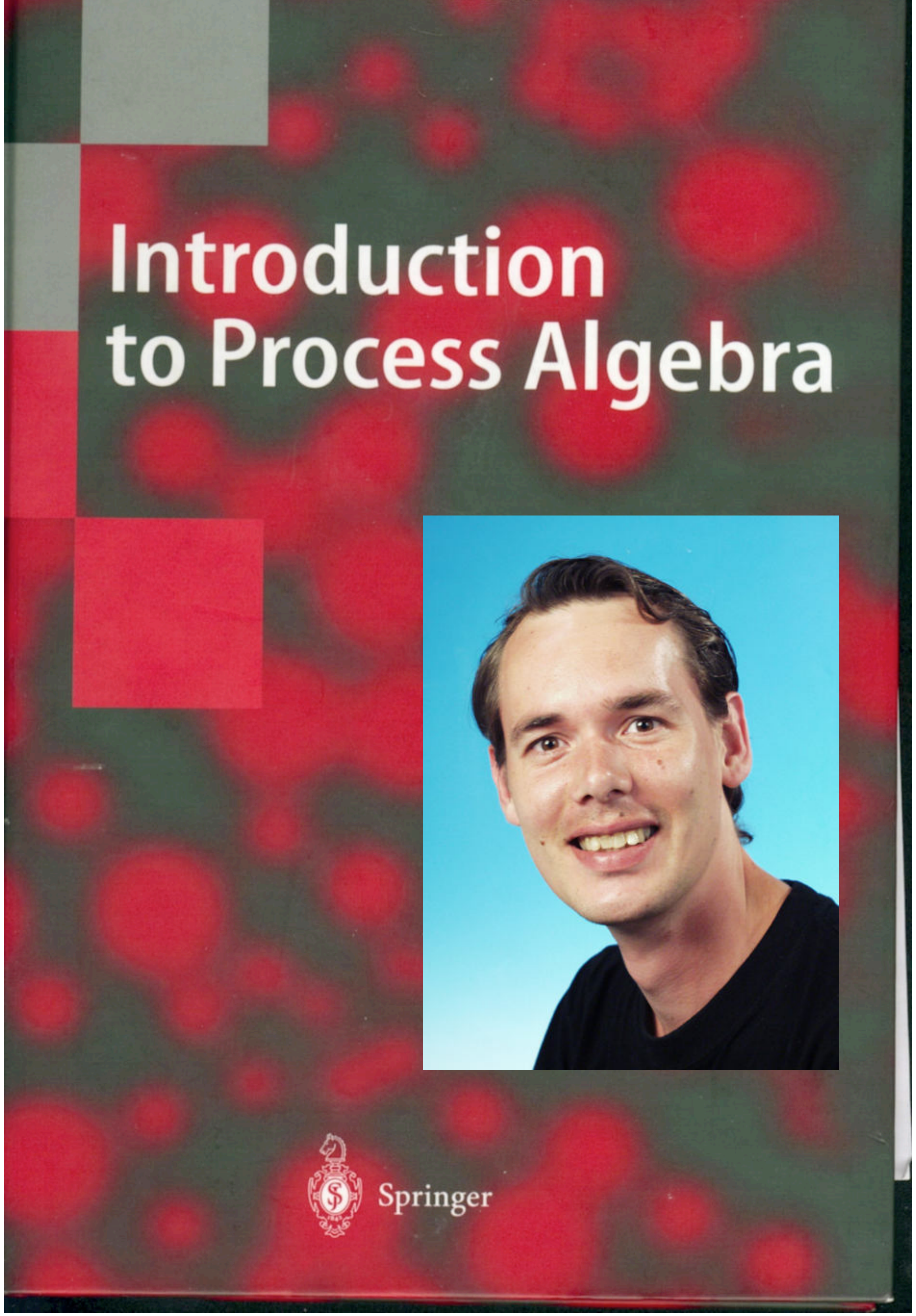
$\alpha = (\sqrt{2})^{\sqrt{2}}$ en $\beta = \sqrt{2}$:
$\alpha^{\beta} = ((\sqrt{2})^{\sqrt{2}})^{\sqrt{2}} = 2$

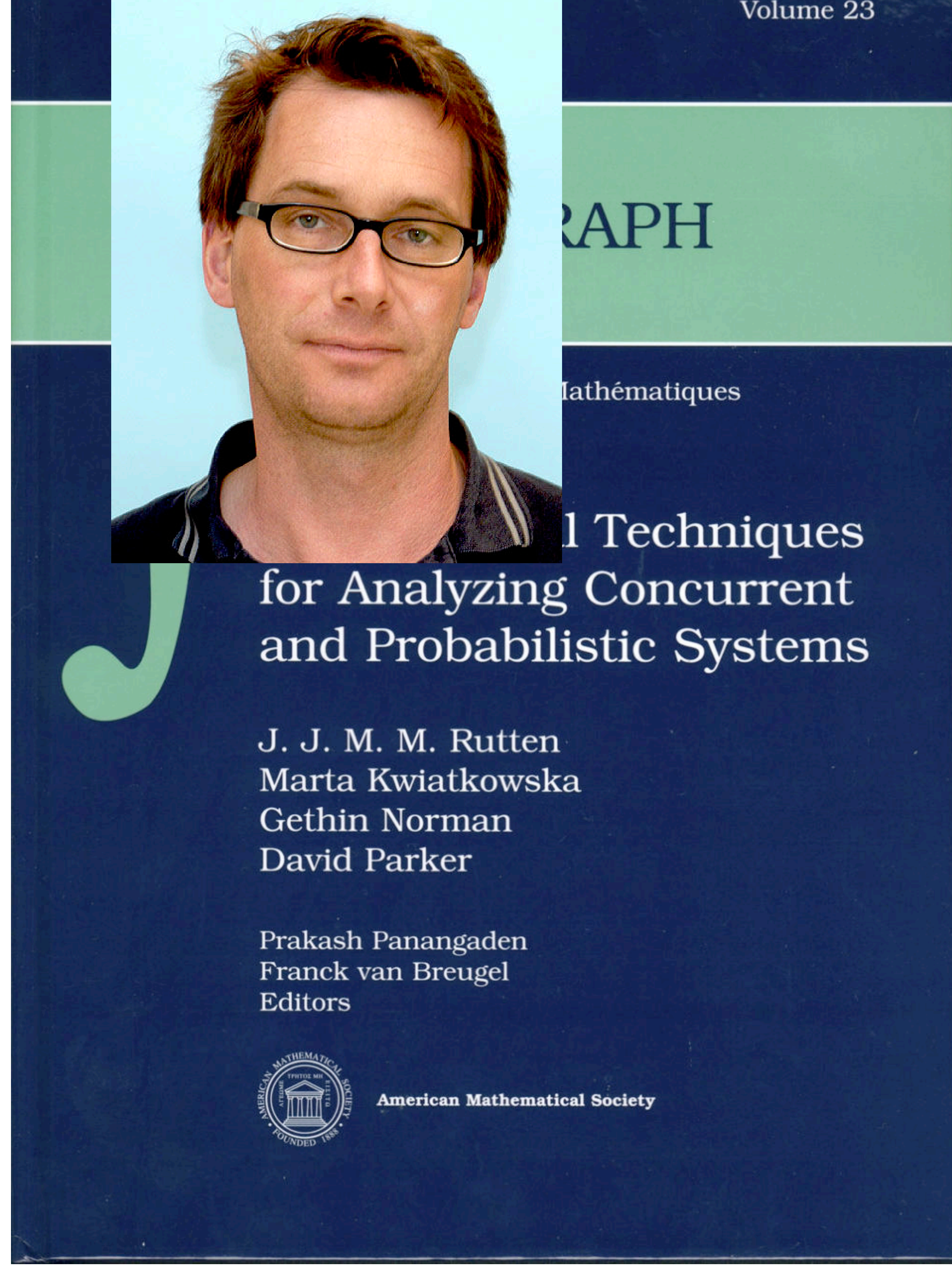
Dana Scott

1646

Robin Milner

1646


Introduction
to Process Algebra
Springer


Volume 23
Techniques
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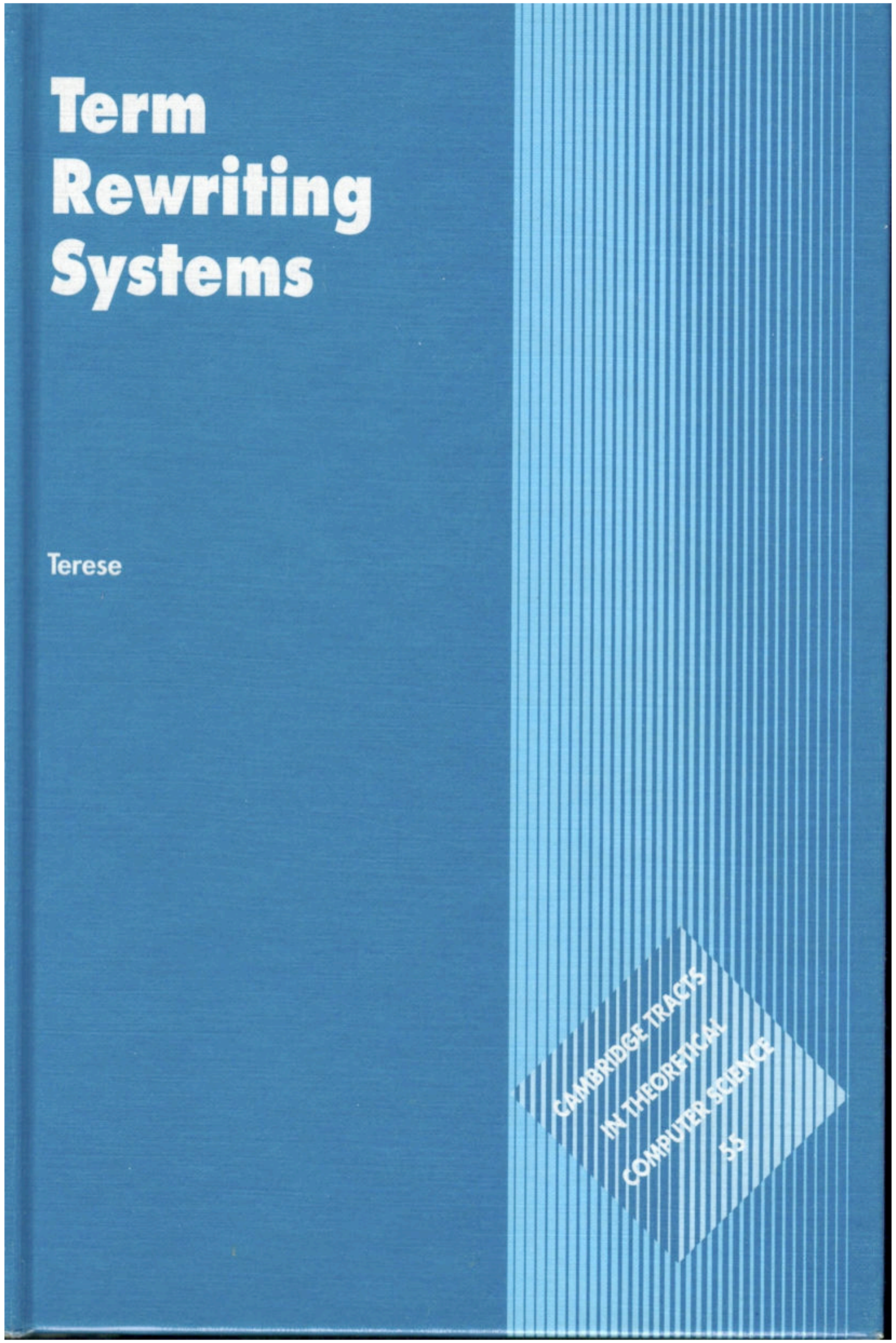
and Probabilistic Systems
J. J. M. M. Rutten
Marta Kwiatkowska
Gethin Norman
David Parker
Prakash Panangaden
Franck van Breugel
Editors
American Mathematical Society



TERESE of the Andes

SII(SII)